Preliminary analysis of tunnel face stability for risk or back analysis

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ABSTRACT: This article develops a fast judging mean of tunnel face stability in naturally complex cases. The result is a «safety factor» allowing design of the stabilizing pressure or back analysis. In urban tunneling projects, this tool can be used in particular along the line as a mean of assessing the likelihood of occurrence of the instability of the face or of the excavation repercussion on the environment. The developed method is a limit equilibrium analysis in continuous medium consisting of the stability assessment of a quarter of ellipsoid ahead of the face. Besides its own weight and the resisting forces along the lateral sides, the volume is subjected to a vertical load, a pressure flow and a counter confining pressure if the latter two are present.

The method is innovative in the failure mechanism considered ahead of the face and in the calculation of the overload: according to the ground conditions, different types of loads, are determined according to the « silo theory ». In the presence of water, undrained and drained conditions, under and above the water level are differentiated. Stratigraphy may also be taken into account with, for each layer, different load geometry and different hydraulic conditions.

Keywords: Face stability, limit equilibrium analysis, silo theory.

1 Introduction

From face bolting and forepoling in conventional methods to pressurization for Tunnel Boring Machine (TBM), the development of face confining technology has allowed the realization of tunnels in increasingly complex geotechnical conditions and under low coverage.

Together with these technological advances, the search for the "just" required confining that allows the stability of the face, limits the deformation without creating uplifts or making impossible the maintenance has been studied theoretically and empirically by different approaches for over 40 years.

The method developed here is a limit equilibrium analysis in continuous medium consisting of the stability assessment of a tetrahedral shape ground volume ahead of the face. Its goal is to achieve a stability factor which is coming from a simplified analysis but allows to consider the combined effects of different actions and approaching a natural complexity.

Besides its own weight and the resisting forces along the lateral faces, the volume is subjected to a vertical load, a pressure flow and a counter confining pressure if the latter two are present.

The geometry of the volume is entirely parameterized by the characteristics of the tunnel cross section (radius and \( r \) or height of the center of vault) and by two angles (opening and slip angle). This «dihedral» is then approximated by the quarter of ellipsoid in which it is included and its stability is analyzed for all pairs of angles.

Overloads applying the dihedral are based on the "theory of silo", but different types of loads but different geometries are analyzed according to the geotechnical conditions encountered. In the presence of water, undrained and drained conditions, under and above the water level are
differentiated. Stratigraphy may also be taken into account with, for each layer, different load geometry and different hydraulic conditions.

2 Geometry, failure mechanism and safety factor

2.1 Geometry of the dihedral wedge in front of the face

The geometry of the volume of ground in front of the face considered in this study is inspired by the shape of polyhedral rocks cut by discontinuities. The dihedral is thus defined by four plane surfaces $S_0, S_1, S_2$ and $S_2'$, and has a plane of symmetry.

Due to the symmetry, the angle of inclination of the sliding edge $\gamma$ and the angle of opening of the frontal face $\beta$ are sufficient to characterize all the surfaces and the volume of the wedge.

The cross section of the tunnel is then approximated by the equilateral triangle $S_0$ and the two inclined side surfaces $S_2$ and $S_2'$ are equivalent. The opening and the length of the upper edge of the front surface are connected to the geometry of the cross section according to the schemes shown below.

2.2 Dihedral equilibrium

The forces acting on the dihedral are shown in figure 3.

Active forces

- $G$ the weight of the wedge with $\omega_0$ density;
- $P_1$ overload provided by the overlying ground;
- $F_0$ horizontal flow force;

Resisting strength

- $\tau_1$ and $\tau_1'$ shear strength of the inclined side surfaces;
- $N_1$ and $N_1'$ normal reactions on both sides of the inclined surfaces;
As the dihedral wedge is symmetrical and as a homogeneous medium is considered, \( \tau_1 \) and \( \tau_1' \) are equal and so are \( N_1 \) and \( N_1' \). A counteractive force \( P_0 \), resulting from the pressure stabilization of the face, can also be taken into account.

### 2.3 Ellipsoidal adjustment

The dihedral wedge can then be approximated by the quarter ellipsoid with semi-axis \( R_1 \), \( R_2 \) and \( R_3 \) in which it is included. The relationship between the volume of the ellipsoid and the volume of the wedge, the side surfaces, the front surfaces and the upper surfaces are expressed in the table adjacent to the figure 4:

<table>
<thead>
<tr>
<th>Volume</th>
<th>Upper surface</th>
<th>Side surface</th>
<th>Front surface</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{\text{ellipsoïde}} )</td>
<td>( S_{1\text{ellipsoïde}} )</td>
<td>( S_{2\text{ellipsoïde}} )</td>
<td>( S_{0\text{ellipsoïde}} )</td>
</tr>
<tr>
<td>( \pi )</td>
<td>( \pi/2 )</td>
<td>( D )</td>
<td>( \pi/2 )</td>
</tr>
</tbody>
</table>

**Figure 4. Adjustment of the dihedral wedge by a quarter of ellipsoid**

The side surfaces of the ellipsoid are approximated by the formula of Knud Thomson. Only the ratio between the lateral surfaces depends on the dimensions of the wedge and of the quarter of ellipsoid. This ratio, denoted \( D \) varies with \( \beta \) between 1.59 and 1.81.

### 2.4 Safety factor

The equilibrium equation of the system in the case of a drained soil and taking into account a flow force is expressed:

\[
S_a \left[ -p_0 \frac{\pi}{2} \right] + S_{01} \left[ \frac{p_0 \pi}{2} \right] + S_i \left[ \begin{array}{c} 0 \\ 0 \\ -p_i \frac{\pi}{2} \end{array} \right] + V_{\text{claire}} \left[ \begin{array}{c} 0 \\ -\omega_i \pi \\ 0 \end{array} \right] + 2S_2 \left[ \begin{array}{c} \sin(\gamma) \cos(i)N - \cos(\gamma)\tau \\ \cos(i)N + \sin(\gamma)\tau \end{array} \right] = 0 \]  

The resolution of the system is carried out by considering an elastic perfectly plastic Mohr-Coulomb criterion and allows expressing the safety factor \( FS \) as the ratio of the active forces on the resisting forces, depending on the angle of inclination of the sliding edge and the opening angle of the frontal face. The system being only governed by the angles \( \gamma \) and \( \beta \), the minimum of safety factor can be determined by varying the angle of inclination of the sliding edge and the opening of the frontal face angle between 0 and 90°.

### 3 Overload for a cohesive-frictional soil

#### 3.1 Geometry of the overload

In the case of a cohesive-frictional soil, the form considered for the overload is a pyramid. This type of mechanism is consistent with the results of 3D finite element models and with those obtained on scale models (cf. Chambon [2] Vermeer [7]). The general expression for the overload is determined from the equilibrium of an elementary slice of ground subject to its own weight \( W \), to the variation of vertical stress \( \partial \sigma_z \) upon its upper and lower faces and to the shear stress \( \tau \) considered on the vertical side faces.
The equation governing the equilibrium depends on the ratio between the perimeter \( \pi \) where the shear force is mobilized and the surface \( S \) of the slice considered and is expressed as:

\[
\frac{d\sigma}{dz} = \frac{\pi}{S} (\tau + w) = 0
\]  

(2)

The relationship between perimeter and area evolves with depth. This report can be expressed as:

\[
\frac{S}{\pi} (z) = B_0 - \lambda z
\]  

(3)

Where \( B_0 \) represents the ratio between the area and the perimeter of the top surface \( S_1 \) of the dihedral and where \( \lambda \) is a shape factor depending on the angle \( \psi \) at the top of the pyramid (cf. figure 6). In the approximation of the diedral wedge by the quarter of ellipsoid, an adjustment factor corresponding to the ratio between the \( B_0 \) of the two volumes is introduced. This factor is constant and equal to \( \sqrt{2} \).

The resolution of the equation allows, assuming a constant ratio between vertical and horizontal stresses over the entire height of the mechanism, to determine the overload \( P_1 \) at \( z = 0 \).

If the depth of the tunnel is sufficient, the pyramid is "closed". Otherwise, the pyramid ends up at the surface. The overload \( P_1 \) is then expressed as the function:

\[
\sigma(z = 0) = P_1 = Max(g_1 \ast P_t + g_2 \ast R \ast wh + g_3 \ast c_1 + g_4 \ast we \ast R;0)
\]  

(4)

Where \( P_t \) is the surface overload at the ground level; \( w \) the wet density of the soil; \( c_1 \) its cohesion and \( we \) the density of water. \( g_1, g_2, g_3 \) and \( g_4 \) are functions depending on the angles \( \beta \) and \( \gamma \), on the internal friction \( \phi \), on the depth of the tunnel and its radius \( R \) and on the groundwater level.

### 3.2 Determination of the opening angle \( \psi \) and of the ratio between horizontal and vertical stresses

The choice of the opening angle of the pyramid \( \psi \) is of prime importance since it determines the height of the AF overload. Different opening angles and different relationships between horizontal and vertical stresses were tested and compared to finite element results.
The study was conducted for a 10 meter tunnel, a unit weight $\gamma$ of 15 kN / m, a tunnel diameter $D$ equal to 10 m, cohesion values varying between 0 and 10 kPa and friction angles varying between 20 and 40°.

The model was validated for an opening angle $\psi$ equal to the friction angle $\varphi$ and for a ration between vertical and horizontal forces equal to the active earth pressure coefficient $K_a$. The graphs in Figure 7 show the evolution of the normalized confining pressure $P_c$ required to obtain a security factor of 1 with the model and the one determined with finite elements calculation by c-phi reduction.

**Figure 7. Evolution of the normalized confining pressure $P_c/\gamma D$ against cohesion $c'$ and friction angle $\Phi'$ for a security factor of 1**

### 4 Overload for a purely cohesive soil

#### 4.1 Geometry of the overload

In order to take into account the flared geometries observed on site or on centrifuged scale models (cf. Kimura [4]), an open cone overload shape as shown in the following figure has been studied.

**Figure 8. Evolution of the normalized confining pressure $P_c/\gamma D$ against cohesion $c'$ and friction angle $\Phi'$ for a security factor of 1**

The “open cone” is defined by two angles: a slump angle $\xi_1$ and an opening angle of the front face $\xi_2$. Equation (2), in the case of purely cohesive soils is expressed:

$$\frac{d\sigma_c}{dz} - \frac{\pi}{S} w = 0$$

Equation (5)

The ratio perimeter on surface as well as lengths $d'$ and $b'$ vary with the depth. Equation (3) is again used and $\xi_1$ and $\xi_2$ angles are parameterized such that: $d'/a_0 = b'/b_0$, $d'/a_{0z} = b'/b_{0z}$.

In purely cohesive soil, stresses are total stresses and break lines reach the surface. Under such conditions the solution of the equation can again be expressed:
\[ P_1 = \text{Max}(g_1 * \text{Pt} + g_2 * R * wh + g_3 * c_u, 0) \]  \(6\)

With: \(\text{Pt}\) surface overload at the ground level; \(wh\) the wet density of the soil and \(c_u\) the undrained cohesion. \(g_1\), \(g_2\), \(g_3\) are functions depending on the angles \(\beta\) and \(\gamma\) and the depth of the tunnel.

4.2 Determination of the opening angles

The adjustment of the opening angles has been carried out by comparing the model results to different other approaches. Methods of Proctor and White, Piaskowski, Ellstein, Brooms and Bennermark as well as analytical limit analysis of Davis, all well adapted to the case of purely cohesive soils were used.

For each model, the ratio between the confining pressure and the vertical stress at the tunnel crown was plotted against the ratio between the depth of cover \(H\) and the tunnel diameter \(D\).

The study was conducted for a tunnel of 10 meters in diameter, a density of 18 kN/m\(^3\), a coverage varying from 5 to 35 m and cohesion values varying between 10 and 50 kPa.

The chosen model is an open cone with opening angles of 13° and without considering shear upon the surface in the plan of the face. Though, in a back analysis this opening angle may be adjusted.

5 Overload in the case of a multi layered ground

The overload on the quarter ellipsoid whose stability is studied varied depending on the characteristics of the overlying ground. The \(P_1\) generic formulation used for both selected models (equations (4) and (6)) can be used to treat the case of a layered medium by adapting the formula and changing the boundary conditions.

To do this, it is necessary to be able to determine the \(B_0\) at the base of each layer and to replace the overload at the surface \(\text{Pt}\) by the overload due to the upper layer.
Overload at the base of the n-layer is expressed as:

\[ P_n = \text{Max} \left( g1 \cdot P_{n+1} + g2 \cdot R \cdot w_h + g3 \cdot c_n + g4 \cdot w_e \cdot R ; 0 \right) \]  

(7)

A recurrence relation is used for determining the \( B0n \) ratio at the n-layered from the ratio between area and perimeter at the base of the underlying \( B0n-1 \) layer and its form factor \( \lambda_{n-1} \).

![Figure 10. Overload scheme in an layered ground](image)

6 Application example

The totality of these developments has been integrated into a software developed by GEOS INGENIEURS CONSEILS.

![Figure 11. Image from the software GEOSTABFRONT](image)

This tool could be used as part of a risk analysis in accordance with the risk management methodology, proposed in the recommendation of the group GT 32 of the French Tunneling Association (AFTES).

The identified risk in this configuration is the instability of the tunnel face. The consequences of the occurrence of the risk vary along the tunnel line according to the presence and the sensibility of the overstructures.

The rapidity in terms of computations with GEOSTABFRONT allows to multiply the number of the calculating sections along the tunnel length. As a consequence, it permits to trace the evolution of the confinement pressure applied to the tunnel face for a set safety factor.
Figure 12. Evolution of the active confinement pressure applied to the tunnel face for a safety factor 1.5.

Urban tunnel project

The likelihood of the occurrence of the tunnel face instability, and by extension the amplitude of surface settlements, is greater when the necessary confinement pressure is stronger. By identifying homogenous zones in terms of consequences, a certain level of risk can be established at any point of the tunnel plot.

7 Conclusions

The model presented in this article is established from a “non-rigorous” explicit analytical method. The constitutive law is thus limited to the knowledge of the strength field of the soil and no information concerning the displacements is available. However, it has been proved that the knowledge of the strain field developed at the tunnel face is essential because the extrusion plays a major role on the development of the surface settlements. The model presents therefore the same limits as the “rigorous” models based on the limit analysis.

The “rigorous” limit analysis methods remain highly effective in simple cases in order to approach the bearable loading area of the problem. In comparison, the potential of this method comes from the simplicity of his approach. It is here possible to consider the combined effects of different actions by singling the geomechanical behaviors.

However, certain limits should be underlined:

- The pressure at the tunnel head is homogenous
- The model does not allow to consider mixt excavation heads
- No vertical flow is added to the overload
- No information concerning the duration of the tunnel head stability is available

The model of the pyramid seems to be particularly efficient in order to treat cases with purely frictional soils. When the cohesion becomes greater, intermediate mechanisms evolving towards the open cone model should be considered. This part of the development is underway.
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References


